S.N.J.B.'s K.K.H.A. Arts, S.M.G.L. Commerce & S.P.H.J. Science College, Chandwad

Department of Mathematics

S.Y.B.Sc. Semester II Paper I MT 221 Linear Algebra

Practical 1: Vector Space, Subspace, Linear Dependence & Independence

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Date: 03/01/2019

Vector Space: (8 Marks Questions)

1. Show that the set $V = \{f | f\ is\ an\ even\ function\}$ with usual addition and scalar multiplication is a vector space. (Apr' 18)
2. Show that the set $V = \{(x, y, 0) | x, y \in \mathbb{R}\}$ with usual addition and scalar multiplication is a vector space. (Apr' 18)

Subspace (4 Marks Questions)

3. Show that the following subsets are subspaces of $\mathbb{R}^3$ under usual addition and scalar multiplication.
    a) $W = \{(x, y, z) | x - y + z = 0\}$ (Apr' 18)
    b) $W = \{(x, y, z) | 2x + 3y - z = 0\}$ (Apr' 17)
4. Determine whether $W = \{(x, y, z) | x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$. (Apr' 18, Apr' 17, Apr'16, Apr'15)
5. Let $M_2(\mathbb{R})$ be real vector space. Let $W = \{A \in M_2(\mathbb{R}) | A\ is\ invertible\ matrix\}$. Is W a subspace of $M_2(\mathbb{R})$? Justify.(Apr' 18)
6. Let $V = M_3(\mathbb{R})$ i.e. the set of all 3×3 matrices with real entries be real vector space. Let $W_1 = \{A \in M_3(\mathbb{R}) | A^t = A\}$, $W_2 = \{A \in M_3(\mathbb{R}) | A^t = -A\}$ Show that W_1 and W_2 are subspaces of $M_3(\mathbb{R})$. (Apr' 18)
7. Let V be a vector space of all real valued functions defined on [0, 1]. Let $W = \left\{f \middle| f\left(\frac{1}{2}\right) = 0\right\}$be the subset of V. Show that W is a subspace of V. (Apr' 15)

Linear Dependence & Independence(4 Marks Questions)

8. Let $\{\bar{u}_1, \bar{u}_2, \bar{u}_3\}$ be linearly independent set of vectors in a vector space V. Let $\bar{v}_1 = \bar{u}_1$, $\bar{v}_2 = \bar{u}_1 + \bar{u}_2$, $\bar{v}_3 = \bar{u}_1 + \bar{u}_2 + \bar{u}_3$. Prove that $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ is linearly independent set.(Apr' 18)
9. Show that the set $\{(1, 1, 0), (1, 1, 1), (1, 0, 0)\}$ is linearly independent. (Apr' 18, Apr' 17, Apr' 16, Apr' 15)
10. Determine whether the set $\{(1, 2, -3), (2, 5, 7), (3, 7, 10)\}$ is linearly independent. (Apr' 17, Apr' 16, Apr' 15)
11. Show that $B = \{1 + 2x + x^2, 2 + x, 1 - x + 2x^2\}$is linearly independent. (Apr' 17)
12. Show that the vectors $\bar{u}_1 = e^x, \bar{u}_2 = cosx, \bar{u}_3 = sinx$are linearly independent in the function space. (Apr' 16, Apr' 15)

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S.Y.B.Sc. Semester II Paper I MT 221 Linear Algebra

Practical 2: Basis, Dimension, Row Space, Column Space & Null

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Date: 17/01/2019

1. Show that $B = \{1 + 2x - 3x^2, 1 - 3x + 2x^2, 2 - x + 5x^2\}$ is basis for real vector space P_2. Also find coordinate vector of $1 + 2x + 3x^2$ relative to this basis.
(8 marks, Apr.18)

2. Determine the dimension and basis for the solution space of the system.
$$x_1 + 2x_2 - 4x_3 + 3x_4 - x_5 = 0$$
$$x_1 + 2x_2 - 2x_3 + 2x_4 + x_5 = 0$$
$$2x_1 + 4x_2 - 2x_3 + 3x_4 + 4x_5 = 0$$
(8 marks, Apr.16)

3. Determine whether the set
   i. $S = \{(1, 2, -3), (1, -3, 2), (2, -1, 5)\}$ (4 marks, Apr.16)
   ii. $S = \{(-1, 2, 3), (2, 5, 7), (3, 7, 10)\}$ (4 marks, Apr.18)
   are basis for $\mathbb{R}^3$.

4. Show that
   i. $B = \{(3, 0, -6), (-4, 1, 7), (-2, 1, 5)\}$ (4 marks, Apr.16)
   ii. $B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ (4 marks, Apr.17)
   iii. $B = \{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$ (4 marks, Apr.17)
   are basis for vector space $\mathbb{R}^3$.

5. Let $\bar{v}_1 = (1,2,1), \bar{v}_2 = (2,9,0), \bar{v}_3 = (3, 3, 4)$. Show that the set $S = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ is a basis for $\mathbb{R}^3$. Find the coordinate vector of $\bar{v} = (5, -1,9)$ w.r.t. S.
(4 marks, Apr.18)

6. Let $\bar{v}_1 = (1,2,3), \bar{v}_2 = (2,3,1), \bar{v}_3 = (3, 1, 2)$. Show that the set $S = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ is a basis for $\mathbb{R}^3$. Find the coordinate vector of $\bar{v} = (9, 4, 11)$ w.r.t. S.
(4 marks, Apr.18)

7. Show that $\{1 + x + x^2, -1 + x, 1 + 2x + x^2\}$ is basis for real vector space P_2.
(4 marks, Apr.18)

8. Show that the set $\{1, t + 1, t^2 + 1\}$ is a basis for P_2. Express $p(t) = t^2 + t + 1$as a linear combination of vectors in that basis.
(4 marks,Apr.18)

9. Let $S = \{e_1, e_2, e_1 + e_2\}$ where $e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)$. Find $L(S)$.
(4 marks, Apr.16)

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10. Show that $S = \{(1,1,0),(1,1,1),(1,0,0)\}$ span $\mathbb{R}^3$. (4 marks, Apr.17)

11. Express $\overline{w} = (5,-12,3)$ as a linear combination of vectors $\bar{v}_1 = (1,2,3)$, $\bar{v}_2 = (-4,5,6)$, $\bar{v}_3 = (7,-8,9)$. (4 marks, Apr.17)

12. Let $W = \{(x,y,z,w) | y+z = 0, x = w\}$ be a subspace of $\mathbb{R}^4$. Find the basis and dimension of $\mathbb{R}^4$. (4 marks, Apr.15, Apr.16)

13. Let $V = M_3(\mathbb{R})$ the set of all 3×3 matrices with real entries be real vector space. Let $W_1 = \{A \in M_3(\mathbb{R}) | A^t = A\}$, $W_2 = \{A \in M_3(\mathbb{R}) | A^t = -A\}$ be subspaces of V. Find basis and dimension of $W_1$and W_2.

(4 marks, Apr.18)

14. Find basis for the subspace spanned by the vectors $(1,1,1),(1,2,3)$ and $(2,3,5)$ of $\mathbb{R}^3$. (4 marks, Apr.15, Apr.16)

15. Find basis and dimension of row space of matrix

(4 marks, Apr.17)

16. Find a basis for the null space, row space and column space of

(8 marks, Apr.18)

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Practical 4: LINEAR TRANSFORMATION I

Date: 21/02/2019

1. Find the range and kernel of the linear transformation $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ defined as $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+z \\ x+y+2z \\ 2x+y+3z \end{bmatrix}$. Also find the rank and nullity of T.

   (Apr. 15, 8 Marks)
2. Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear map with $T\bar{e_1} = \bar{e_2}, T\bar{e_2} = \bar{e_3}$ *and* $T\bar{e_3} = 0$, where $\bar{e_1} = (1,0,0), \bar{e_2} = (0,1,0)$ *and* $\bar{e_3} = (0,0,1)$. Show that $T \neq 0, T^2 \neq 0$ *but* $T^3 = 0$. (Apr. 18, 4 Marks)
3. Find domain and codomain of $T_2 \circ T_1$ and find $T_2 \circ T_1(x,y)$ if $T_1(x,y) = (2x,3y), T_2(x,y) = (x-y,x+y)$. (Apr. 18, 4 Marks)
4. Let $T_1: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ and $T_2: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be a linear transformation given by

   $$T_1(x,y) = (x+y,y), T_2(x,y) = (2x,y,x+y)$$

   Find formula for $T_2 \circ T_1$. (Apr. 16, 4 Marks)
5. Compute $T_2 \circ T_1(x,y,z)$ if $T_1(x,y,z) = (x-y,y+z,x-z)$, $T_2(x,y,z) = (0,x+y+z)$. (Apr. 15, 16, 18, 4 Marks)
6. Verify rank-nullity theorem for linear transformation $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ given by $T(x,y,z) = (x+y-z, x-2y+z, -2x-2y+2z)$.

   (Apr. 15, 16,17, 18, 8 Marks)
7. Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ given by $T(x,y,z) = (x+y+2z, x+z, 2x+y+3z)$. Find Kernel and image of T. (Apr. 16, 8 Marks)
8. Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be defined by $T(x,y,z) = (x+3y+z, x+2, x+y-z)$. Determine whether T is a linear transformation. (Apr. 17, 4 Marks)
9. If $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$is defined by $T(x,y,z) = (2x-3y-3z, x+z, y+z)$ then show that T is a linear transformation. (Apr. 17,18, 4 Marks)
10. Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation defined by $T(x,y,z) = (x,y,x+y+z)$. Find Ker (T). (Apr. 15,17, 4 Marks)
11. Find rank of the linear transformation $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ given by $T(x,y,z) = (x+y-3z, 2x-2y+z, -2x-2y+6z)$. (Apr. 17, 4 Marks)

12. Let $A = \begin{bmatrix} -1 & 2 & 1 & 3 & 4 \\ 0 & 0 & 2 & -1 & 0 \end{bmatrix}$ and let $T: \mathbb{R}^5 \longrightarrow \mathbb{R}^2$ be a linear transformation such that $T_A(X) = AX$. Find $T(1,0,-1,3,0)$. (Apr. 16, 4 Marks)

13. Let $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear map such that $T(1,1) = (0,2), T(1,-1) = (2,0)$. Find formula for $T(x,y)$, hence compute $T(-2,1)$.

14. $T: \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ be the linear transformation given by the formula $T(x_1, x_2, x_3, x_4) = (4x_1 + x_2 - 2x_3 - 3x_4,\ 2x_1 + x_2 + x_3 - 4x_4, 6x_1 - 9x_3 + 9x_4)$. Show that $(3,-8,2,0) \in KerT\ and\ (0,0,6) \in R(T)$. (Apr. 18, 4 Marks)

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Practical 5: LINEAR TRANSFORMATION II

Date: 28/2/2019

1. Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be multiplication by a matrix A. Determine whether T has an inverse; if so, find $T^{-1}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ where $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

2. For the linear transformation $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ given by
$$T(x, y, z) = (2x + y - z, 3x - 2y + 4z)$$
Find the matrix A of T with respect to bases $B_1 = \{(1,1,1), (1,1,0), (1,0,0)\}$ and $B_2 = \{(1,3), (1,4)\}$ of $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively.

3. Find the matrix of linear transformation $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ given by
$$T(x, y, z) = (-x - y + z, x - 4y + z, 2x - 5y)$$
with respect to the standard basis of $\mathbb{R}^3$ and $\mathbb{R}^3$.

4. Let $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation defined by $T(x, y) = (x + y, x + y)$ find matrix of T with respect to standard basis of $\mathbb{R}^2$.

5. Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear transformation defined by
$$T(x, y, z) = (x + y + z, y + z)$$
Find matrix A of T with respect to standard bases of $\mathbb{R}^3$ *and* $\mathbb{R}^2$ respectively.

6. Find the basis for the kernel of the linear transformation $T: P_2 \longrightarrow P_3$ given by $T(p(x)) = x \cdot p(x)$.